Analysis of a Pneumatic Isolation System for Inertial Instrument Testing

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The dynamic behavior of the pneumatically-suspended isolation platform at the U.S. Air Force Academy Guidance and Control Laboratory was investigated. A computer simulation model was used to evaluate the effect of changing physical parameters of the pad suspension system, and to determine the requirements for improving the present configuration. Results of this study provided the motivation for incorporating an adjustable two-way flow valve between the upper and lower air chambers. This allowed setting the damping factor at a point which would optimize pad response, so as to minimize the peak amplification of the pad transfer function.

Nomenclature

A = amplitude of pad motion
B = damping coefficient
B_{OPT} = optimum damping coefficient
C = restoring force gain
F_d = disturbance force
F_r = restoring force
K = spring constant of air cylinder
M = mass of seismic pad
N = ratio of air volume of upper to lower chambers
W_d = damped natural frequency
W_n = natural frequency
W_{PK} = frequency of minimum peak amplitude ratio
W_0 = natural frequency with no damping
W_∞ = natural frequency with infinite damping
X_1 = motion of the seismic pad
X_3 = motion of the base slab

Introduction

THIS paper describes an investigation aimed at analyzing and simulating the behavior of the Inertial Test Facility at the U.S. Air Force Academy's Guidance and Control Laboratory. The system investigated consists of a 450,000 lb seismic isolation mass supported by twenty pneumatic springs which are actively controlled in height by mechanically actuated servo valves. The system was originally designed by Barry Controls and much of the analysis consists of an extension of Barry's theoretical analysis to this specific installation and mission.

The investigation was aimed at answering three specific questions. The first was: "How does the isolation pad behave?" This includes the development of a reasonable analytic model that would describe the dynamic behavior of the pad resulting from both force disturbances on the pad and base motion disturbances. The second question, aimed at the specific mission of the facility, was: "What is the desired behavior of the pad?" Identification of unacceptable performance responses, based on analytical and experimental studies, revealed the undesirable characteristics of the pad. The third question was: "What can be done to improve the pad dynamics?" This was directed first at the effects of the parameters over which we had most control, e.g., damping between chambers of the pneumatic suspension. It also included the possibility of the use of such approaches as an active tilt feedback mechanization and other more elaborate methods of stabilization.

The objective of this study was to determine how much pad performance could be improved using methods at hand, and to develop an analytic model which could be used to evaluate more advanced methods of pad stabilization.

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Description of the Isolation Pad and Monitor System

Isolation Pad

The isolation platform is constructed of a steel reinforced concrete block which appears as a 25 ft^2 from the top, and has a cruciform shape from the bottom. Nine piers, poured as an integral part of the block, protrude through the floor and provide the usable work area. The total pad weight of approximately 450,000 lb is supported by twenty pneumatic isolators. When floated, the block is approximately $\frac{1}{8}$ in. above the base slab.

The base slab which supports the pneumatic isolators is 24-in. reinforced concrete. This slab is physically distinct from the concrete basement floor around it, and rests on 3 ft of compacted aggregate fill. The subgrade is a native, granular, cohesionless, decomposed granite material.

Seismic Monitor Equipment

The isolation platform monitoring system, consisting of two tilt transducers and three seismometers, is used to measure the relative angle of tilt about two axes perpendicular to the gravitational force field and the acceleration factors associated with vibration in the horizontal and vertical axes. The system utilizes two mercury pool, dual-capacitance level sensors manufactured by Teledyne-Geotech. The moving-coil, short-period seismometers are used to detect the acceleration and displacement of the isolation pad motion. The minimum detectable signal from these sensors is 0.1×10^{-6} g and they have a frequency response of ± 3 db from 1 to 50 Hz. The tiltmeters have a resolution of 0.01 arc sec and a frequency response flat to ± 3 db from d.c. to 0.06 Hz.

Mathematical Model

Single Axis Model

The dynamic behavior of the height-controlled isolation pad can be modeled in the single-axis case as a mass-spring-damper system as shown in Fig. 1. The spring constant K of an air spring is inversely proportional to the volume of air contained in that pneumatic cylinder. The factor N is equal to the ratio of the volume of air in the lower chamber to that of the upper chamber. The damping coefficient B in a pneumatic isolation system is varied by changing the size of the orifice between the two chambers.

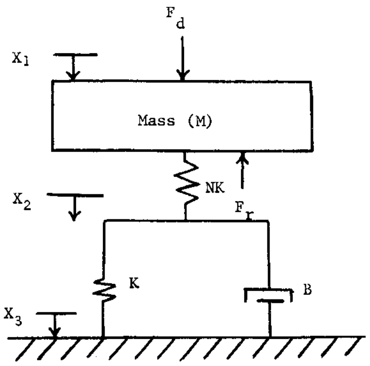

Fig. 1 Simplified linear model.

The restoring force controlled by a height sensing valve is represented by F_r and the disturbance forces exerted on the pad are represented by F_d. An additional source of pad disturbance X_3 is cause by motions of the base slab. Writing the summation of forces at points X_1 and X_2 gives the following equations:

$$M\,d^2X_1/dt^2 = NK(X_2 - X_1) + F_d - F_r \tag{1}$$

$$NK(X_1 - X_2) = K(X_2 - X_3) + B\,d(X_2 - X_3)/dt \tag{2}$$

Differentiating Eq. (1) and substituting into Eq. (2) gives

$$d^3X_1/dt^3 + [K(N+1)/B]\,d^2X_1/dt^2 + (NK/M)\,dX_1/dt + (NK^2/BM)X_1 = (NK/M)\,dX_3/dt + (NK^2/BM)X_3 - (1/M)\,d(F_r - F_d)/dt - [K(N+1)/BM](F_r - F_d) \tag{3}$$

Taking Laplace transforms

$$\left(s^3 + \frac{K(N+1)}{B}s^2 + \frac{NK}{M}s + \frac{NK^2}{BM}\right)X_1(s) = \left(\frac{NK}{M}s + \frac{NK^2}{BM}\right)X_3(s) - \left(\frac{1}{M}s + \frac{K(N+1)}{BM}\right)[F_r(s) - F_d(s)] \tag{4}$$

For this system, the restoring force F_r can be represented by a force which is proportional to the integral of the pad displacement X_1 with respect to the base slab X_3. The integral relationship is present due to the fact that height variations mechanically vary the opening of a two-way valve controlling air flow to the cylinders. This flow rate over a finite time period changes the cylinder pressure which results in a net change in total force on the pad.

$$F_r(s) = (C/s)\,[X_1(s) - X_3(s)] \tag{5}$$

Substituting (5) into (4), multiplying through by s, combining terms and solving for $X_1(s)$ gives

$$X_1(s) = \frac{1}{\Delta}\left(\frac{NK}{M}s^2 + \frac{NK^2 + CB}{BM}s + \frac{CK(N+1)}{BM}\right)X_3(s) + \frac{s}{\Delta}\left(\frac{1}{M}s + \frac{K(N+1)}{BM}\right)F_d(s) \tag{6}$$

where

$$\Delta = s^4 + \frac{K(N+1)}{B}s^3 + \frac{NK}{M}s^2 + \frac{NK^2 + CB}{BM}s + \frac{CK(N+1)}{BM}$$

Determination of Physical Parameters

In order to estimate the value of the physical parameters given in Eq. (6) it was necessary to make some experimental response measurement. By putting a step disturbance input of approximately 160 lb on the pad and recording the outputs of a seismometer and a tiltmeter mounted on the pad, the typical traces shown in Fig. 2 are obtained.

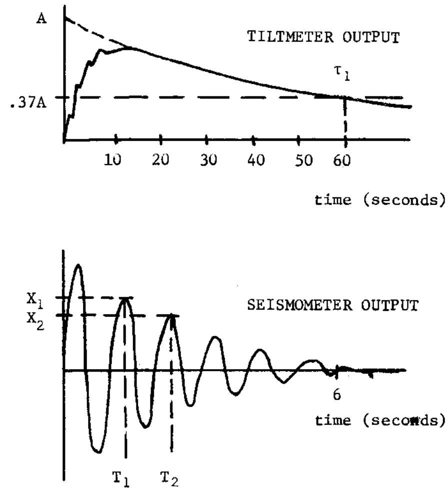

Fig. 2 Single axis step response.

From the seismometer trace, the damped natural frequency W_d and damping factor ζ can be determined assuming a second-order response

$$W_d = 2\pi/(T_2 - T_1) \quad \text{(rad/sec)} \tag{7}$$

$$W_n = W_d/(1 - \zeta^2)^{1/2} \tag{8}$$

$$\zeta = [1/W_n(T_1 - T_2)\ln(X_2/X_1) \tag{9}$$

From experimental data $f_d = W_d/2\pi \cong 1$ cycle-sec and $\zeta \cong 0.1$. The mass m is calculated to be 14,650 slugs from the total weight and the ratio of volumes N is 6. If the measured response is represented by a second-order system, the following relationships hold

$$W_n^{\,2} = K_{eq}/M; \quad 2\zeta W_n = B/M \tag{10}$$

where

$$K_{eq} = [N/(N+1)]K$$

From Eq. (10) the following parameters can be determined:

$$K_{eq} = M(2\pi f_n)^2 = (14{,}650)(4)(\pi^2)(1) = 580{,}000 \text{ lb/ft}$$

$$B = 4\pi\zeta f_n M = (4)(\pi)(0.1)(1)(14{,}650) = 18{,}400 \text{ lb/ft/sec} \tag{11}$$

$$K = [(N+1)/N]K_{eq} = (\tfrac{7}{6})(580{,}000) = 677{,}000 \text{ lb/ft}$$

The restoring force gain C is determined from the tiltmeter trace. The time constant τ_1 required to restore the pad to its initial position is related to C in the following manner.

From Eq. (5), assuming that $X_3(s) = 0$

$$F_r(s) = (C/S)X_1(s)$$

$$F_r(t) = C\int_0^t X_1(\tau)\,d\tau \tag{12}$$

Ignoring the 1 Hz frequency oscillations which quickly die out, the amplitude of the experimental trace $X_1(t)$ can be represented as an exponential function with amplitude A and time constant τ_1

$$X_1(t) = A\,e^{-t/\tau_1} \tag{13}$$

Substituting (13) into (12) and integrating

$$F_r(t) = C\int_0^t A\,e^{-\tau/\tau_1}\,d\tau = CA[-\tau\,e^{-\tau/\tau_1}]_0^t \tag{14}$$

Evaluating at $t = \infty$, where the restoring force becomes equal to the disturbing force

$$F_r(\infty) = F_d = CA\tau_1 \tag{15}$$

Therefore

$$C = F_d/A\tau_1$$

But F_d/A is the spring stiffness K_{eq}. Therefore,

$$C = (K_{eq}/\tau_1) \quad \text{lb/ft/sec}$$

$$C = 580{,}000/60 \text{ sec} \approx 10{,}000 \text{ lb/ft sec} \tag{16}$$

Determination of Optimum Damping

Analytical Methods

The effect of varying the damping factor B by changing the size of the flow restriction between the two air chambers can be determined by analyzing the analytical transfer function between pad and floor motion. For the case where $B = C = 0$ (valve open; zero damping), Eq. (6) reduces to

$$\frac{X_1}{X_3} = \frac{NK/(N+1)M}{s^2 + NK/(N+1)M} \tag{17}$$

The behavior of such a system would be represented by a mass on an undamped spring having a natural frequency of

$$W_0 = [NK/(N+1)M]^{1/2} \tag{18}$$

For the case where $B = \infty$ (valve closed; infinite damping), Eq. (6) reduces to

$$\frac{X_1}{X_3} = \frac{NK/M}{s^2 + NK/M} \tag{19}$$

The behavior of this system would also be represented by a mass on an undamped spring having a natural frequency of

$$W_\infty = (NK/M)^{1/2} \tag{20}$$

Thus, an infinite amplitude ratio response occurs for both extreme settings of the damping valve when forced at their respective natural frequencies. The ratio of these two natural frequencies is

$$W_\infty/W_0 = (N+1)^{1/2} = 2.65 \tag{21}$$

For intermediate valve settings, some damping will result. To determine the effect of this damping on reducing the amplitude ratio of the pad transfer function, the calculated values for M, N, K, and C were substituted into Eq. (6). The magnitude of the resulting function can be written as

$$\left|\frac{X_1}{X_3}\right| = \frac{1}{D}\left[\left(-276W^2 + \frac{3.23\times10^6}{B}\right)^2 + \left(\frac{186\times10^6}{B}W\right)^2\right]^{1/2} \tag{22}$$

where

$$D = \left[\left(W^4 - 276W^2 + \frac{3.23\times10^6}{B}\right)^2 + \left(\frac{-4.74\times10^6}{B}W^3 + \frac{186\times10^6}{B}W\right)^2\right]^{1/2}$$

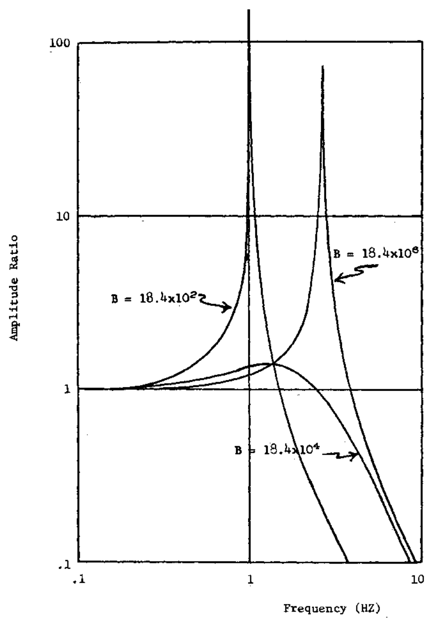

Fig. 3 Pad frequency response for various values of B.

Figure 3 shows a plot of $|X_1/X_3|$ as a function of W for values of B from 184 to 18,400,000. Examination of this figure indicates that all of the transfer functions pass through a common point. The frequency at which this point occurs can be determined by equating the magnitudes of Eqs. (17) and (19). Solving for W, we obtain

$$W = W_{PK} = [2NK/M(N+2)]^{1/2} = 8.34 \text{ rad/sec} = 1.33 \text{ Hz} \tag{23}$$

Substituting this value of W into Eq. (22) for any value of B gives

$$|X_1/X_3| = (2+N)/N = 1.33 \tag{24}$$

Since all transfer function curves pass through this common point, the smallest peak amplification that can be achieved with this system is 1.33.

In order to determine the value of B that gives this minimum peak amplitude ratio, Eq. (22) can be differentiated with respect to W and the result set equal to zero. Evaluating the result at the value of W_{PK} given in Eq. (23) and solving for the optimum value of B gives

$$B_{\text{OPT}} = [KM(N+1)(N+2)/4N]^{1/2} = 216{,}000 \text{ lb/ft/sec} \tag{25}$$

An alternate way of examining the response of this system as a function of the damping parameter B is to use the root locus method of factoring the characteristic equation, Δ, given in Eq. (6)

$$s^4 + \frac{K(N+1)}{B}s^3 + \frac{NK}{M}s^2 + \frac{NK^2+CB}{BM}s + \frac{CK(N+1)}{BM} = 0 \tag{26}$$

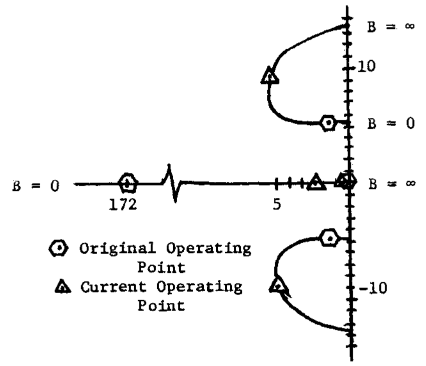

Fig. 4 Roots of characteristic equation as a function of B.

Figure 4 shows these root locations as B varies from 0 to ∞. By observing the eigen values for a few sample values of B we can approximate the general trend of response. There will always be a slow exponential type response for realistic values of B, but this mode will be relatively unimportant. This is due to the fact that in both floor and disturbance transfer functions there will be a zero at nearly the same location. A second mode will be a second-order response with $0 < \zeta < 0.7$ and $0.9 < f_n < 2$. This will normally be the dominant mode of response. The third mode is an exponential form whose importance varies throughout the range of B. For higher values of B this mode may be as slow as the damping envelope of the second-order mode and with a reasonably large magnitude.

Experimental Methods

The analytic study of the original isolation system, as well as experimental data, indicated a peak transmission of eight to ten between pad and floor motion which was considered unacceptable. In view of this the original pad design was modified to include a variable flow valve between chambers on the basis of the analytic study. This valve then gave the capability of changing the parameter B to obtain a more satisfactory response. Although it was not possible to immediately correlate valve settings and values of B, it was known that a closed valve corresponded to $B = \infty$ and an open valve to $B = 0$. It was desired to determine an "optimum" valve setting which would correspond to the B_{OPT} determined earlier. To obtain this information a test mass was placed on and off the pad while recording the seismometer outputs. From these seismometer traces the peak acceleration, damped natural frequency, and estimate of cycles to damp were recorded. Some of this information is given in Fig. 5. These curves indicate that a valve setting around 0.25 (that is, 0.25 of a turn from the point where flow between the chambers was just starting) would nearly minimize both peak acceleration and damping. The damped natural frequency recorded was assumed

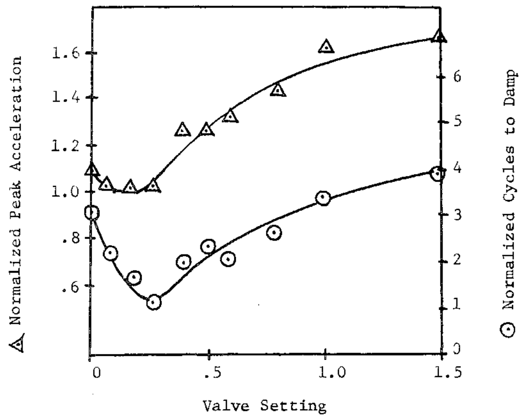

Fig. 5 Normalized experimental response characteristics.

to be that of the dominant second-order mode discussed earlier in this section. By using the same information as that of Fig. 4 (the roots of the characteristic equation as a function of B) it was possible to correlate a valve setting to a value of B through the damped natural frequency (Fig. 6). Looking at this figure it can be seen that the value of B_{OPT} determined analytically corresponds to a valve setting of slightly more than 0.2, which agrees closely with the valve setting chosen by examining the system time responses. Thus, this was chosen as the best setting, given the present operating conditions.

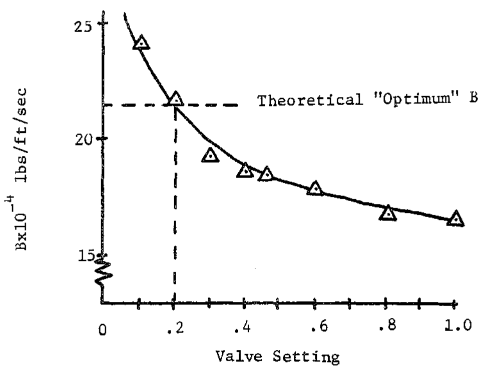

Fig. 6 Damping factor vs valve setting as experimentally determined.

Analog Computer Simulation

In order to examine the time response of the pneumatic isolation system to both base motion disturbances and force disturbances on the pad for different values of the physical parameters, an analog computer model of the system was created.

This was originally done on an EAI TR-20 analog computer, but as the model grew it was transferred to an Applied Dynamics AD-4 hybrid computer which was utilized mostly in its analog mode. The equations of motion mechanized were Eqs. (1) and (2) which describe the motion of a linear mass, spring, damper combination as shown in Fig. 1 with an integral restoring force proportional to flow valve opening. The classical method of equal coefficients was used as a first estimate of maximum values expected for a 160 lb step input of force on the pad. This resulted in normalizing values for X_1 of approximately 0.1×10^{-3} in. and of 0.1×10^{-3} in. for X_2. The motion of the floor was modeled as a sinusoidal forcing function with W varying from 0.1 to 40 Hz with an amplitude of 0.1×10^{-3} in. During frequency response testing of the model it was necessary to reduce the amplitude of this input as the natural frequency of the pad model was approached.

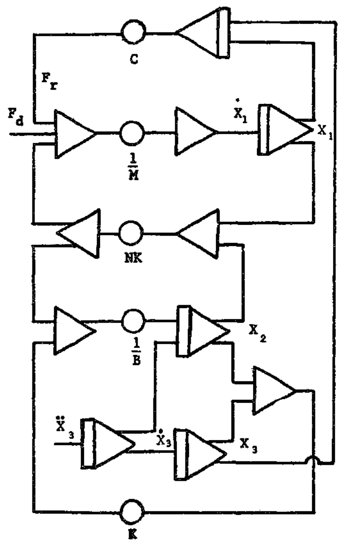

Fig. 7 Analog computer flow diagram.

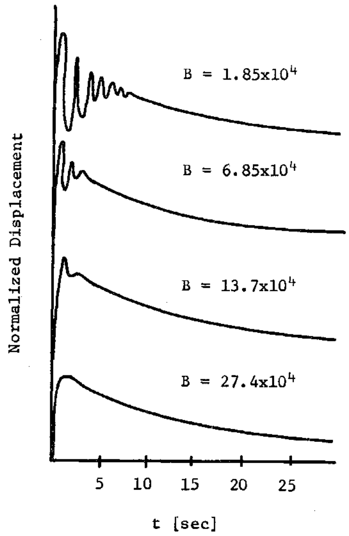

Fig. 8 Time response to step force disturbance for various values of B.

The qualitative flow diagram used is shown in Fig. 7. Although not the most efficient network, this was chosen because it allowed each physical parameter to be changed individually. This was useful both for initially matching experimental data and later for checking the effect of damping coefficient changes on system response. This model simulated the motion at only one of the three sensor locations and was used to verify the experimentally calculated values for K, B, and C. Fine adjustments to these values were made on the computer to match the experimental step response. At this point it was necessary to run the analog signal X_1 through a first-order filter with a break frequency of about 0.1 Hz in order to match actual tiltmeter readings in shape. This was due to the filtering effect of the mercury pool tiltmeters being used. The single axis model was also used to investigate the effect of changing the damping between the two air cylinders. The value of B was increased from its original value to twenty times that value. The effect of this on frequency response curves confirmed the results obtained earlier in the digital computer studies of Sec. IV. Figure 8 shows some of the normalized time response curves of displacement due to a step disturbance on the pad. It can be seen that a response with no overshoot would occur between curves C and D, that is, in the region of the analytically calculated B_{OPT}. This analog computer simulation model will prove useful in the design and verification of a closed loop active tilt control system.

Conclusions and Recommendations

Two major results have emerged so far from this investigation. First, by using a linear analytic model of the pad, it is possible to create an analog computer model which simulates actual pad response over the lower frequency range.

Secondly, it was shown that the lower frequency pad dynamics can be appreciably altered by varying the damping between the air chambers. This has resulted in the incorporation of a variable damping capability at the U.S. Air Force Academy facility. Experimental verification of the computer results has been accomplished and a greatly improved damping factor B equal to 216,000, has been selected for this system.

The next logical step to be accomplished in the improvement of this seismic isolation system is to incorporate a gravity-stabilized, tilt feedback control system. This will involve the use of an electronic tilt indicator, electrical-to-pneumatic transducer, and a compensation network. The analog simulation model will provide an excellent means for verifying the design of this system and for optimizing its performance.